ISB Senior Contest 2002 Team Questions

1. A 2-digit number has a 1 attached to its right, making a 3-digit number.

   When the old number is subtracted from the new, the difference is 208.

   What was the original number?

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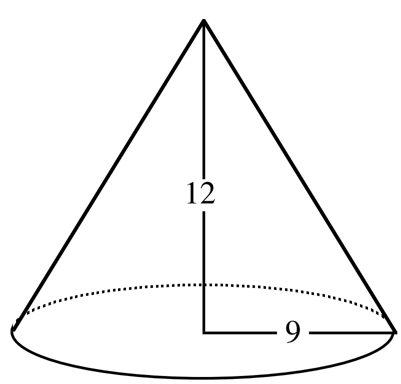
2. A right circular cone has radius of base = 9 and height = 12, as shown.
   Find the radius of a sphere which has a volume equal to the cone's. (Round to nearest 0.01)



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4. The Very High Ladder has an infinite number of equally-spaced steps.

   Jack B. Nimble jumps up the ladder, 13 steps at a time.

   After the first jump of 13 steps he falls back 1 step.
   After the second jump of 13 steps he falls back 2 steps. After the third he falls back 3 steps, and so on.

   After how many jumps will Jack fall back to the ground?

5. A triangle has sides of 10, 24, and 26.

   Find the radius of the inscribed circle. (nearest 0.1)

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NO CALCULATOR

6. Solve for all θ on $0° \le \theta \le 90°$: $1 - \sin(2\theta) = \cos(4\theta)$

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NO CALCULATOR

7. Find the smallest positive integer k for which $1260k = N^3$, where N is an integer.

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NO CALCULATOR

8. Given $a = \log_{10} 8$ and $b = \log_{10} 9$, express $\log_{10} 15$ in terms of a and b, and simple arithmetic as required.

9. Find the mean **and** standard deviation of this set of scores: 96, 91, 87, 84, 81, 80, 76, 76, 72, 67.

   (Round answers to nearest 0.1)

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10. Given: $\sqrt{92}$ is a leg of a right triangle.

    Find two **integers** which make the other leg and hypotenuse.

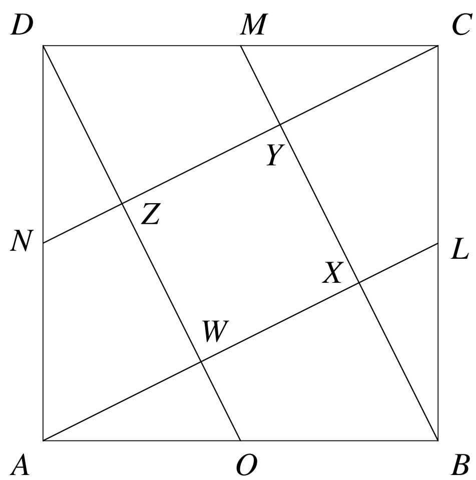
11. The sum, S_n, of n terms of an arithmetic progression is $2n + 3n^2$.

    Find and simplify an expression for the r^{th} term, a_r, in terms of r.

12. A "Major Year" is one in which the sum of the numbers formed by the first two digits and the last two digits equals the number formed by the middle two digits. For example, 1538 was a Major Year, because 15 + 38 = 53.

    Find the next Major Year after 2002.

13. Points *L*, *M*, *N*, *O* are midpoints of the sides of square *ABCD*. The area of *ABCD* is 100. Find the area of quadrilateral *WXYZ*



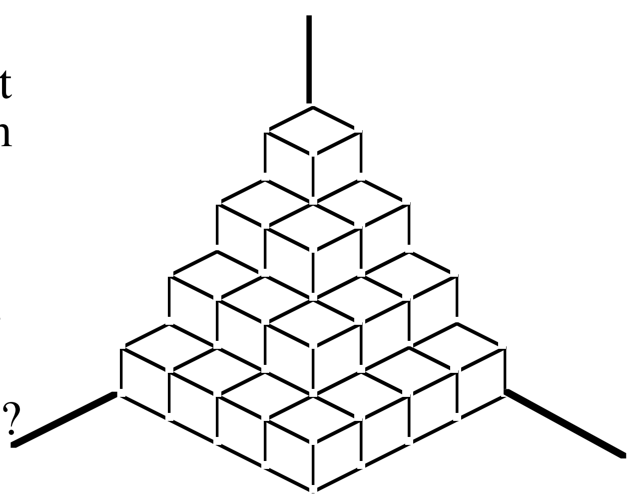
NO CALCULATOR

14. If $\left(r + \frac{1}{r}\right)^2 = 3$, then $r^3 + \frac{1}{r^3} =$

15. Santa Claus has eight reindeer pulling his sleigh, arranged in four rows of two each. Dancer and Prancer must be side-by-side. Donder and Blitzen cannot be side-by-side. In how many different ways can Santa arrange his reindeer?

16. This stack of "blocks" sits in a corner. It has been built from toothpicks which form the edges of the "blocks".

 How many toothpicks are there in the stack, including the ones hidden inside, on the bottom, and in the back?



17. Find all complex number solutions for z in the equation: $z^2 = -8i$. (Express in the form $a + bi$.)

18. Matthew can beat Jeff by $\frac{1}{10}$ mile in a 2-mile race. Jeff can beat Steven by $\frac{1}{5}$ mile in a 2-mile race.

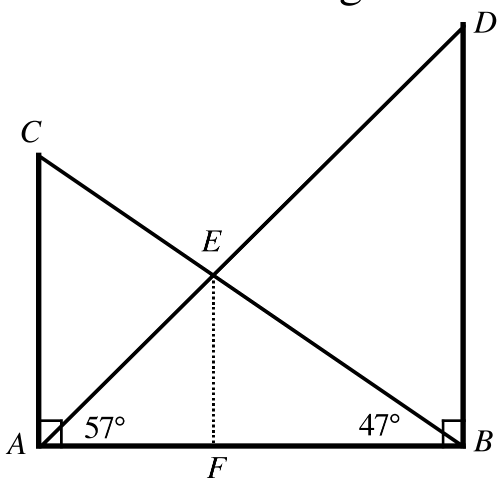
    Assume all runners run at a constant speed.

    By what distance could Matthew beat Steven in a 2-mile race? (Exact answer in decimal or fraction form)

19. A sequence $\{a_n\}$ is defined as follows:
$a_1 = 2;\ a_2 = 3;\ a_n = (a_{n-2})(a_{n-1})$ for $n \geq 3$.

In the prime factorization of a_{13}, what is the power of 3?

20. A street *AB* is between walls *AC* and *BD*.
Wires *AD* and *BC* make angles of 57° and 47° with the street and cross at *E* as shown.
BD is 5 meters higher than *AC*.



Find *AB*, the width of the street.

(Round to nearest 0.1 m)