

“A Flight of Fancy”

International School Bangkok Senior Mathematics Competition

Saturday, 23 March, 2002

Individual Contest

Do not open this question paper until told to do so.

There are 25 multiple-choice questions.

Time limit = 90 minutes

Scoring: Right answer = + 4 points
 Wrong answer = – 1 point
 Blank = 0 points

Some questions offer the choice
... E) none of these

Choose this if you determine that none (or more than one) of answers A, B, C, or D are correct, or if there is insufficient or inconsistent information given in the question. In the scoring, this choice counts as an answer and may be right or wrong.

You may use **one** calculator (with programs cleared) and ruler, but no protractor, compasses, or other instrument, translating dictionaries, electronic devices, or other aides.

Scrap paper will be provided. You may write on this question paper. It is yours to keep.

1. At every hour (1:00, 2:00, 3:00, ... and so on throughout 24 hours every day) a plane leaves New York for Los Angeles. Simultaneously a plane leaves LA for NY. The trip takes 5 hours each way. How many planes from LA will a plane from NY pass in the air during its flight? (Don't count planes landing or taking off at the same time.)
- A) 4 B) 5 C) 6 D) 9 E) 10
2. The arithmetic mean of a set of 50 numbers is 38. Two numbers of the set, namely 45 and 55, are discarded. What is the arithmetic mean of the set of numbers remaining?
- A) 38.5 B) 37.5 C) 37 D) 36.5 E) 36
3. If $x, 2x + 2, 3x + 3$ is a geometric progression, the 4th term is
- A) -27 B) $-13\frac{1}{2}$ C) 12 D) $13\frac{1}{2}$ E) 27
4. The top, front, and side of a rectangular solid have areas of 20.25, 18, and 12.5. What is its volume?
- A) $41\bar{6}$ B) 48 C) 50.75 D) 67.5 E) 72
5. Simplify $\left(1 - \frac{1}{3}\right)\left(1 - \frac{1}{4}\right)\left(1 - \frac{1}{5}\right) \dots \left(1 - \frac{1}{n}\right)$
- A) $\frac{1}{n}$ B) $\frac{2}{n}$ C) $\frac{2(n-1)}{n}$ D) $\frac{2}{n(n+1)}$ E) $1 - \frac{2}{n!}$
6. If $\log_{10} m = b - \log_{10} n$, then $m =$
- A) $\frac{b}{n}$ B) bn C) $10^b \cdot n$ D) $\frac{10^b}{n}$ E) $b - 10^n$

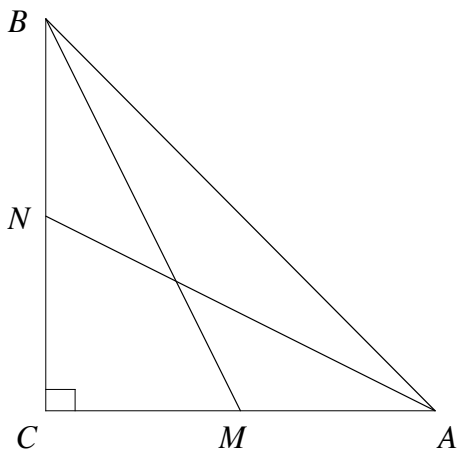
7. The line $2x + 5y = -10$ is reflected over the line $y = x$. The image is then reflected over the x -axis. That image is then reflected over the y -axis. What is the equation of the final image?

A) $5x + 2y = -10$ B) $5x + 2y = 10$ C) $5x - 2y = 10$
 D) $2x + 5y = 10$ E) $2x - 5y = -10$

8. The area of an ellipse is πab , where a and b are the semi-axes of the ellipse. Find the area of this ellipse: $x^2 + 3y^2 + 8x + 18y + 7 = 0$

A) $6\sqrt{3}\pi$ B) $12\sqrt{3}\pi$ C) 36π D) $30\sqrt{3}\pi$ E) $\frac{29\sqrt{3}}{3}\pi$

9. $BM = \sqrt{40}$, $AN = 5$. BM and AN are medians. Find the length of AB .

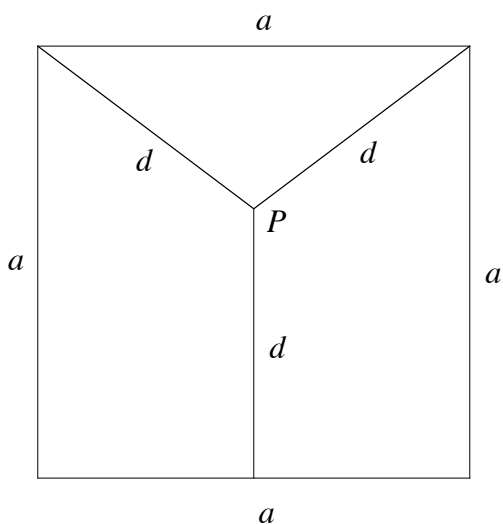


A) $2\sqrt{15}$
 B) $5\sqrt{2}$
 C) $\sqrt{30}$
 D) $2\sqrt{13}$
 E) none of these

10. If x men working x hours per day for each of x days produce x articles, how many articles (not necessarily an integer) are produced by y men working y hours a day for each of y days?

A) $\frac{x^3}{y^2}$ B) $\frac{y^3}{x^2}$ C) $\frac{x^2}{y^3}$ D) $\frac{y^2}{x^3}$ E) y

11. The sides of the square have length a . Point P is distance d from two vertices and a side, as shown. Find an expression for d in terms of a .



- A) $\frac{3a}{5}$
 B) $\frac{5a}{8}$
 C) $\frac{3a}{8}$
 D) $\frac{a\sqrt{2}}{2}$
 E) $\frac{a}{2}$

12. Given $A = \begin{bmatrix} -2 & 2 \\ -3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & -6 \\ 7 & -7 \end{bmatrix}$.

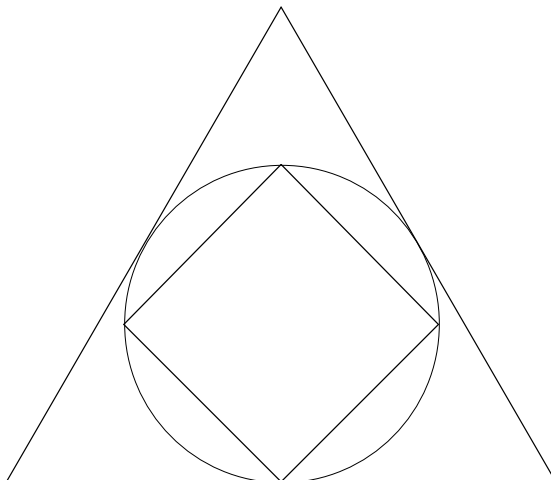
Solve the equation $A \cdot X = B$.

- A) $\begin{bmatrix} 1 & -2 \\ -3.5 & 0 \end{bmatrix}$ B) $\begin{bmatrix} 2 & -10 \\ -2 & -4 \end{bmatrix}$ C) $\begin{bmatrix} 2.5 & 1.5 \\ 3 & 1 \end{bmatrix}$
 D) $\begin{bmatrix} -3 & 1 \\ -8 & 5 \end{bmatrix}$ E) none of these

14. In a game at Sanuk Fair, a player throws three darts at a large rotating wheel. About 30% of the wheel's area has coloured sections. If a dart hits a coloured section the player wins a prize. Assume the darts hit the wheel's surface at random. What is the probability that a player wins exactly two prizes? (Choose nearest answer.)

- A) 6% B) 9% C) 19% D) 27% E) 36%

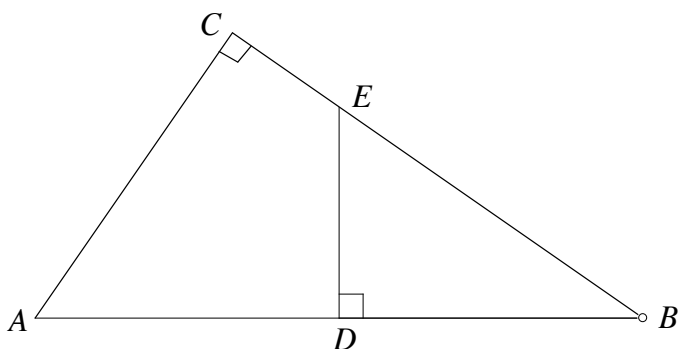
15. A club with x members is organized into four committees according to these two rules:
 (1) Each member belongs to exactly two committees;
 (2) Each pair of committees has exactly one member in common.
 Then $x =$
- A) 4 B) 6 C) 8 D) 16 E) cannot be determined
16. Each angle of a regular polygon is 15° more than each angle of a regular 15-sided polygon. How many sides does the polygon have?
- A) 9 B) 17 C) 20 D) 30 E) 40
17. A circle is inscribed in an equilateral triangle and a square is inscribed in the circle.



The ratio of the area of the triangle to the area of the square is

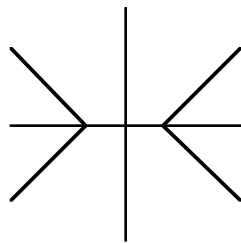
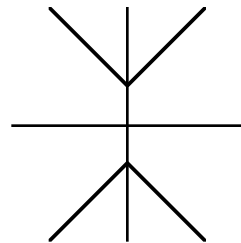
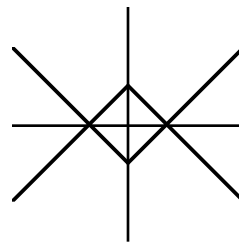
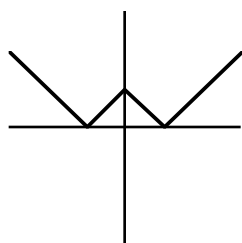
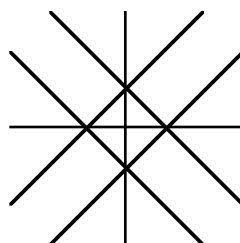
- A) $\sqrt{3}:1$ B) $\sqrt{3}:\sqrt{2}$ C) $3\sqrt{3}:2$ D) $3:\sqrt{2}$ E) $3:2\sqrt{2}$
18. Thirty-one books are arranged from Left to Right in order of increasing prices. The first book costs at least \$1. The price of each book differs by \$2 from that of each adjacent book. For the price of the book at the extreme right a customer can buy the middle book and an adjacent one. Which statement must be true?
- A) the adjacent book referred to is at the left of the middle book
 B) the middle book sells for \$36
 C) the cheapest book sells for \$4

- D) the most expensive book sells for \$64
E) none of these
19. Solve this equation for complex number z : $z - 40 = 3iz$
A) $5 + 15i$ B) $-5 - 15i$ C) $4 + 12i$ D) $-4 - 12i$ E) none of these
20. If x and y are both integers, how many ordered pairs (x, y) satisfy the equation $(x - 8)(x - 10) = 2^y$?
A) 0 B) 1 C) 2 D) 3 E) more than 3
21. In solving a problem leading to a quadratic equation, one student makes a mistake in the constant term and obtains 8 and 2 for the roots. Another student makes a mistake in the coefficient of the first-degree term and finds -9 and -1 for roots. The correct equation should have been
A) $x^2 - 10x + 9 = 0$ B) $x^2 + 10x + 9 = 0$ C) $x^2 - 10x + 16 = 0$
D) $x^2 - 8x - 9 = 0$ E) none of these
22. Singapore is very near the equator. A plane leaves Singapore and flies 15,000 km on a straight course to City A. Similarly, a plane leaves Singapore and flies 15,000 km to City B. The circumference of the Earth is 40,000 km. What is the probability that A and B are within 15,000 km of each other? Choose the nearest answer.
A) $\frac{1}{6}$ B) $\frac{1}{3}$ C) $\frac{1}{2}$ D) $\frac{2}{3}$ E) 1
23. $AD = DB = 10$. $AC = 12$.
Find the area of quadrilateral $ADEC$



- A) 75
- B) $58\frac{1}{2}$
- C) 48
- D) $37\frac{1}{2}$
- E) none of these

24. What shape is the graph of $|x| - |y| = 1$?

- A) 
- B) 
- C) 
- D) 
- E) 

25. If 2137^{753} is multiplied out, the units digit in the final product is

- A) 1
- B) 3
- C) 5
- D) 7
- E) 9